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ON ELECTRONS WITH ENERGIES GREATER THAN 1600 KEV

IN THE EARTH'S RADIATION BELTS¹

A. I. Yershovich

The concept that fluxes of electrons with a kinetic energy $E_k \geq 1600$

/1457

Kev can result from the decay of fast neutrons in the albedo of the earth's atmosphere was first expressed by Nakada (Ref. 1). His calculations refer to an "average" electron with energy ~ 300 Kev (in the system whose origin is at the center of mass) which flies out at an angle $\theta_e =$

90° with respect to the direction of the moving neutron. It is necessary, however, to consider the angular and energy distribution of electrons.

Let us determine the probability W , that the total energy of the electron in the laboratory system $E_L \geq E_0$ if the velocity of the neutron is equal to V_0 . The subscripts "L" and "C" refer respectively to the quantities of the laboratory system of reference and the system of mass center. We shall measure the energy of the electron in units of $m_0 c^2$ where m_0 is the mass of the electron at rest. In terms of these units the maximum energy associated with the origin of electrons in the C-system is equal to 2.53.

We let

$$\beta_0 = V_0/c, \gamma_0 = (1 - \beta_0^2)^{-1/2}, \beta = v/c$$

where v is the velocity of the electron. Then

$$E_L = \gamma_0 E_C (1 + \beta_0 \beta_c \cos \theta_c) \quad (1)$$

¹Translated from Izvestiya Akademii Nauk SSSR, Seriya Geofizicheskaya, No. 9, 1457-1461, 1963.

We determine the maximum angle $\theta_{c \max}(E_c)$ for the emission of an electron of energy E_c so that $E_L \geq E_o$:

$$\cos \theta_{c \max} = \frac{\frac{E_o}{\gamma_o E_c} - 1}{\beta_o \beta_c} \quad (2)$$

By letting $\theta_c = 0$ in equation (1) we obtain the lowest energy of the electron E_c , for which $E_L = E_o$:

$$E_{c \min} = \frac{E_o}{\gamma_o(1 + \beta_o \beta_c)} \quad (3)$$

Because $E_c \beta_c = (E_c^2 - 1)^{1/2}$, $\gamma_o \beta_o = (\gamma_o^2 - 1)^{1/2}$

we have

$$E_{c \min} = E_o \gamma_o - (E_o^2 - 1)^{1/2} (\gamma_o^2 - 1)^{1/2} \quad (4)$$

The function for the distribution of electrons in the C-system is isotropic and has the form given in Reference 2,

$$F(E_c) = \frac{0.614}{4\pi} (E_c^2 - 1)^{1/2} E_c (2.53 - E_c)^2 \quad (5)$$

where

$$\int_0^{4\pi} d\Omega_c \int_0^{2.53} F(E) dE = 1 \quad (6)$$

$d\Omega_c = \sin \theta_c d\theta_c d\phi_c$ - is an element of the solid angle in a spherical system of coordinates whose axis coincides with the direction of motion of the neutron. The unknown probability $W(E_o, \gamma_o)$ is equal to:

$$W(E_o, \gamma_o) = 2\pi \int_{E_{c \min}}^{2.53} F(E) dE \int_0^{\theta_{c \max}} \sin \theta d\theta \quad (7) \quad \underline{1458}$$

where $\theta_{c \max}$ and $E_{c \min}$ are determined from (2) and (4). If we let

$$I_n = \int_{E_c \min}^{2.53} (E^2 - 1)^{1/2} E^n dE. \quad K_m = \int_{E_c \min}^{2.53} E^m dE \quad (8)$$

we obtain

$$W(E_0, \gamma_0) = 0.307 \{ 2.53^3 I_1 - 5.06 I_2 + I_3 + (\gamma_0^2 - 1)^{-3/2} (5.06 E_0 K_2 - 2.53^3 E_0 K_1 - E_0 K_3 + 2.53^3 \gamma_0 K_2 - 5.06 \gamma_0 K_3 + \gamma_0 K_4) \}. \quad (9)$$

The flux of fast neutrons of the albedo with kinetic energy E_n (Mev) at a geocentric distance r (measured in terms of the earth's radii) is equal to

$$j_n = \frac{A E_n^{-B}}{r^2} \text{ 1/cm}^2 \text{ sec Mev} \quad (10)$$

Hess showed (Ref. 3) that in the region from 10 Mev to 1 Bev $A = 0.8$, $B = 2$. Lenchek and Singer (Ref. 4) obtained $A = 2$, $B = 1.8$. Measurements of the energy spectrum of neutrons in the upper atmosphere do not make it possible to show preference to either form of the spectrum.

The number of electrons $n(E_0, \lambda_0, \lambda)$ with energies $E_L \geq E_0$ generated during the β -decay of neutrons (in cm^3 per second) along the geomagnetic line of force which intersects the surface of the earth at latitude λ_0 , is equal to

$$n(E_0, \lambda_0, \lambda) = \frac{\cos^4 \lambda_0}{c t_n \cos^4 \lambda} \int_{\gamma_0 \min}^{\infty} X(\gamma_0, A, B) d\gamma_0 \quad (11)$$

where $t_n \approx 1100$ sec is the average life of a neutron.

$$X(\gamma_0, A, B) = A \cdot 0.40^{1-B} (\gamma_0 - 1)^{-B} (\gamma_0^2 - 1)^{-1/2} W(E_0, \gamma_0) \quad (12)$$

To find $\gamma_0 \min$ we let $E_c = 2.53$ and $\theta_c = 0$

$$\gamma_0 \min = \frac{E_0}{2.53(1 + \beta_0 \beta_c)} \quad (13)$$

Because $\gamma_0 \beta_0 = (\gamma_0^2 - 1)^{1/2}$ we have

$$\gamma_{0 \min} = 2.53E_0 - (2.53^2 - 1)^{1/2}(E_0^2 - 1)^{1/2}. \quad (14)$$

Let us assume that $E_0 = 4.13$ which corresponds to the kinetic energy of electrons $E_k = 1600$ Kev. Then $\gamma_{0 \min} = 1.139$. Consequently, electrons with energies greater than 1600 Kev can be formed during the decay of neutrons with energies E_n greater than 130 Mev.

The functions X_1 and X_2 corresponding to different energy spectra of neutrons (according to Refs. 3 and 4) are represented in Figure 1 for $E_0 = 4.13$. The axis of abscissas shows the values of the kinetic energy of the neutron $E_n = 940 (\gamma_0 - 1)$ instead of γ_0 . For convenience the values of X_1 are increased by a factor of 10. From Figure 1 we find that

$$\int_{\gamma_{0 \min}}^{\infty} X_1 d\gamma_0 \approx 3 \cdot 10^{-4}, \quad \int_{\gamma_{0 \min}}^{\infty} X_2 d\gamma_0 \approx 3 \cdot 10^{-8} \text{ (1/cm}^2 \text{ sec)}$$

For the energy spectrum of neutrons in the form given by (Ref. 4) ($A = 2$, $B = 1.8$) equation (11) reduces to:

$$n(\lambda_0, \lambda) = 9.1 \cdot 10^{-17} \frac{\cos^4 \lambda_0}{\cos^4 \lambda}. \quad (15)$$

The function $n(\lambda_0, \lambda)$ is shown in Figure 2 for $\lambda_0 = 50^\circ$ and 60° (curves a and b respectively). /1459

The equatorial value of the flux of electrons captured by the geomagnetic field can be determined quite accurately from the equation: (see figures 3 and 7 of Ref. 5).

$$j_0 = n(\lambda_0, \lambda = 0^\circ) v T, \quad (16)$$

where T is the life of electrons whose image points are near the

equatorial plane. As shown by Tverskoy (Ref. 6) $T = 10^8 E_K^{3/2} N_0^{-1}$ where

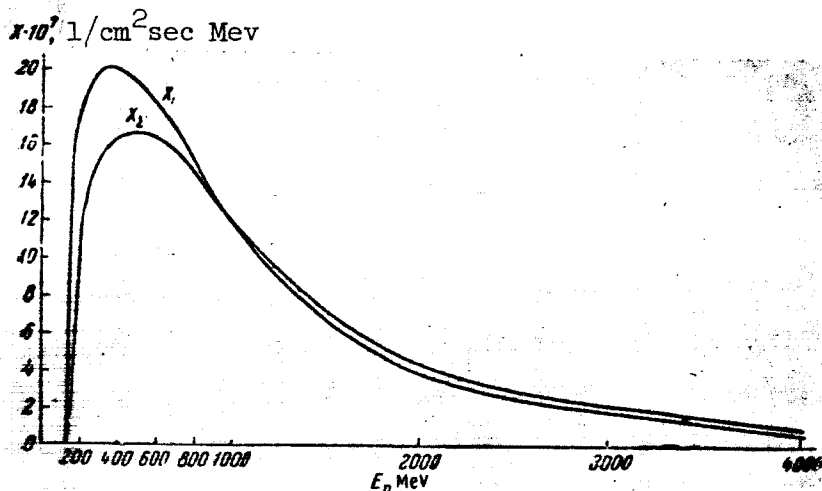


Figure 1.

E_K is the kinetic energy of electrons in Kev, N_0 is the concentration of the cold plasma at the respective altitudes. For the lines of force which have $\lambda_0 = 60^\circ$ and 50° , we may take N_0 to have corresponding values of $1.5 \cdot 10^2$ and $9 \cdot 10^2 \text{ cm}^{-3}$ (Ref. 4). Consequently, on the lines of force $\lambda_0 = 60^\circ$ and 50° the fluxes of electrons with energies in excess of 1600 Kev may reach values of $7.3 \cdot 10^3$ and $3.3 \cdot 10^3 \text{ 1/cm}^2 \text{ sec}$ respectively. If the energy spectrum of the neutrons has the form given in (Ref. 3) these values of the fluxes should be decreased by a factor of 10. A comparison with experimental data in the region where the radiation of the external zone is a maximum (Ref. 7) makes it possible for us to assume that electrons with $E_K \geq 1600 \text{ Kev}$ can be formed directly by the decay of fast neutrons in the atmosphere.

We shall show that in this case the fluxes and electrons must have substantial anisotropy, and the majority of the particles move at large pitch angles ξ_0 at the equator.

From equation (5) we obtain a distribution function for the electrons in the laboratory system of reference (see Ref. 8).

$$f(\gamma_0, E_L, \theta_L) = \frac{0.614}{4\pi} \gamma_0 E_L (E_L^2 - 1)^{1/2} (1 - \beta_0 \beta_L \cos \theta_L) [2.53 - \gamma_0 E_L (1 - \beta_0 \beta_L \cos \theta_L)]^2 \quad (17)$$

This function is represented on the polar diagram (Figure 3) to an arbitrary scale for $E_L = 4.13$: curve a is constructed for $\gamma_0 = 1.6$

($E_n = 564$ Mev), curve b is constructed for $\gamma_0 = 2$ ($E_n = 940$ Mev). The

distribution of electrons is quite anisotropic especially for large neutron energies E_n . For example, the maximum angles, at which electrons

fly out, in the L-system are $\theta_{L \max} = 35.5, 34.5, 29.5$ and 22.5° for neu-

tron energies $E_n = 564, 940, 1880$ and 3760 Mev respectively (it follows

from expression (17) that $\cos \theta_{L \max} = (\beta_0 \beta_L)^{-1} - 2.53(\gamma_0 \beta_0 E_L \beta_L)^{-1}$).

We shall therefore assume that the majority of electrons are emitted in the approximate direction of neutron movement. Neutrons with rather large energies (according to Figure 1 they are primarily responsible for formation of electrons which are of interest to us) are emitted by the

earth's atmosphere at a zenith angle close to 90° (Ref. 9). Thus the directions from which the neutrons arrive at a point P_λ of latitude λ

form a cone with the apex at point P_λ . Since the earth's sphere is

inscribed within this cone it is easy to determine the apex angle ω

$$\sin \omega = \frac{1}{r} = \frac{\cos^2 \lambda_0}{\cos^2 \lambda} \quad (18)$$

The angle α between the axis of the cone and the direction of the magnetic field \vec{H} at the point P_λ is given by the expression

$$\operatorname{tg} \alpha = \frac{H_\lambda}{H_r} = \frac{1}{2} \operatorname{ctg} \lambda. \quad (19)$$

At low latitudes α is large; consequently, the majority of the electrons are injected at large pitch angles ξ_λ . This distribution of electrons according to pitch angles at the time of their origin is retained at higher latitudes because as the angle α is decreased there is a simultaneous increase in the cone angle 2ω . We should therefore expect the equilibrium function for the distribution of captured electrons according to pitch angles at the equator $j_0(\xi_0)$ to have a maximum at $\xi_0 = 90^\circ$.

The form of the function $j_0(\xi_0)$ is determined, for example, using the method described in Reference 10 if we know the distribution of electrons with respect to the magnetic field vector \vec{H} at each point P_λ along the line of force at the time of injection. This distribution $\Phi(P_\lambda)$ can be obtained from expression (17) if we tie angles θ_L and φ_L of a spherical coordinate system, in which the polar axis coincides with the direction of neutron motion, with angles ξ and η of the spherical coordinate system whose axis is directed along the vector \vec{H} .

It is easy to show that the angle $\psi(\delta)$ between the polar axes of both coordinate systems is given by the expression

$$\cos \psi(\delta) = \sin \alpha \sin \omega \cos \delta + \cos \alpha \cos \omega \quad (20)$$

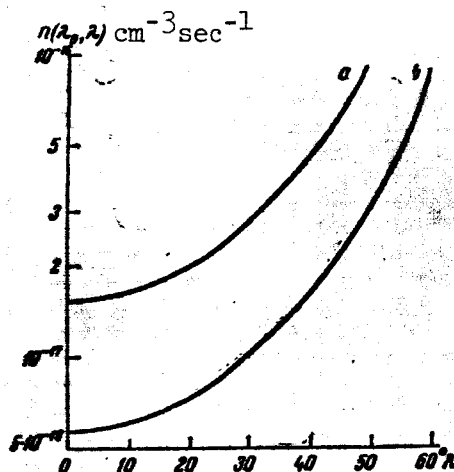


Figure 2.

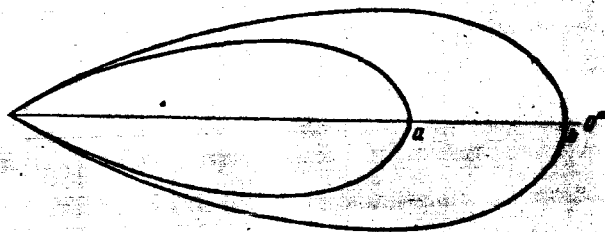


Figure 3.

where ω and a are the parameters of the point P_λ (see Refs. 18 and 19) while δ is the azimuth of the resulting cone (i.e., the direction of neutron motion) which is measured between the plane passing through the axis of the cone and the direction of the magnetic field \vec{H} at the point P_λ . Then

$$\cos \theta_L = \sin \psi(\delta) \sin \xi \cos \eta + \cos \psi(\delta) \cos \xi \quad (21)$$

$$\operatorname{tg} \phi_L = -\cos \psi(\delta) \operatorname{ctg} \eta + \sin \psi(\delta) \operatorname{ctg} \xi \operatorname{cosec} \eta \quad (22)$$

Let us assume that the number of neutrons arriving at point P_λ does not depend on the azimuth δ . Averaging out over δ we get

$$\Phi(a, \omega, E_L, \xi, \eta) = \frac{1}{2\pi} \int_{r_0}^{\infty} dr_0 \int_0^{2\pi} \frac{f(r_0, E_L, \theta_L) \sin \theta_L d\theta_L d\phi_L}{\sin \xi d\xi d\eta} d\delta \quad (23) \quad \underline{1461}$$

In the right side of expression (23) the angular quantities must be expressed in terms of ξ and η by means of equations (20) - (22).

The unknown function Φ of electron distribution at the time of origin does not have azimuthal symmetry with respect to the direction of the earth's magnetic field \vec{H} .

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